

Solutions of the problems

1. Problem proposed by M. Hosszú (Miskolc):

Solve the functional equation

$$(1) \quad f(x+y-xy) + f(xy) = f(x) + f(y),$$

where f is a real valued function defined on real numbers.

M. Hosszú solved equation (1) under the assumption of differentiability.

H. Świątak ([3]) proved the following theorem 1:

If a solution of equation (1) is continuous either at the point $x = 0$ or at the point $x = 1$ or at the points $x = a$, $x = a + \frac{1}{a} - 1$ it has to have the form

$$(2) \quad f(x) = Ax + B,$$

where A and B are arbitrary constants.

Z. Daróczy ([1]) assumed that the function f is integrable on the interval $(0, 1)$ and proved that also in this case the solutions of equation (1) have form (2).

Next it was shown by H. Świątak ([4]) that the assumption of the existence of the integral $\int_0^{\varepsilon} f(t) dt$ for some $\varepsilon > 0$ implies satisfying the assumptions of theorem 1.

The most general solution of equation (1) in the domain of functions isn't known. It was shown only (cf. H. Świątak [3]) that if the function $\varphi(x) = f(x) - f(0)$ is asymmetrical, equation (1) can be reduced to Jensen's functional equation.

The most general solution of equation (1) in the domain of distributions was given by I. Fenyő ([2]). It has form (2) but neither results of paper [3] nor those of papers [2] and [4] are special cases of the result of Fenyő (They imply that every locally integrable solution of equation (1) has form (2) only almost everywhere).

REFERENCES

- [1] Z. Daróczy, *Über die Funktionalgleichung $f(xy) + f(x + y - xy) = f(x) + f(y)$* , Publ. Math., Debrecen (to appear).
- [2] I. Fenyő, *On the general solution of a functional equation in the domain of distributions*, Aequationes Math. (to appear).
- [3] H. Światak, *On the functional equation $f(x + y - xy) + f(xy) = f(x) + f(y)$* , Matematički Vesnik 5 (20), (1968), 177-182.
- [4] H. Światak, *Remarks on the Functional Equation $f(x + y - xy) + f(xy) = f(x) + f(y)$* , Aequationes Math., Vol. 1, No 3 (1968), 239-241.