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On C^r solutions of the Böttcher equation

We shall consider the Böttcher equation

$$\varphi[f(x)] = \{\varphi(x)\}^p, \quad p > 1. \tag{1}$$

For a fixed number a, $0 < a \leqslant \infty$ we denote by U^a (for every real a) the class of functions which are defined and positive in $(0, a)$ and for which there exists $\lim_{x \to 0+} x^{-a} f(x)$ and the limit is positive.

In paper [1] M. Kuczma has proved under some assumptions on the function $f(x)$ that there exists in $\langle 0, a)$ exactly one solution of equation (1) that belongs to the class U^a.

Giving the example with the function $f(x) = x^2$ and $p = 2$, M. Kuczma has also proved the lack of uniqueness of C^r solutions of equation (1) in $\langle 0, a)$.

It turns out, however, that this is a general situation.

We denote by H^r the following set of hypotheses:

$$(H^r) \quad \begin{cases} f \in U^p,\ p > 1,\ f(x) \text{ is of class } C^r \text{ in } \langle 0, a),\ 0 < a \leqslant \infty, \\ f'(x) > 0 \text{ and } f(x) \neq x \text{ in } (0, a) \text{ and } x^{-p} f(x) \text{ (defined for} \\ x = 0 \text{ as its limit) is of class } C^r \text{ in } \langle 0, a). \end{cases}$$

Theorem 1. Suppose that hypotheses (H^1) are fulfilled. Then there exists a C^1 solution of equation (1) in $\langle 0, a)$ depending on an arbitrary function.

Theorem 2. Suppose that hypotheses (H^r) $r \geqslant 1$, are fulfilled and $r < p$. Then there exists a C^r solution of equation (1) in $\langle 0, a)$ depending on an arbitrary function.

REFERENCES

[1] M. Kuczma, *Sur l'équation fonctionnelle de Böttcher*, Mathematica, Cluj 8 (31), 2, (1966), 279-285.