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Sets of Covering Mappings

1. Let M and P denote p -dimensional separable topological manifolds; P is assumed to be connected and ϱ will denote a metric function on P . Let $\mathcal{C} = \mathcal{C}(M, P)$ be the set of all continuous mappings from M into P with the so-called C^0 -Whitney topology: if $f \in \mathcal{C}$ then the sets

$$V(f, \delta) = \{g \in \mathcal{C} : \varrho(f(x), g(x)) < \delta(x)\}$$

(when $\delta : M \rightarrow R_+$ is a positive continuous function) form a neighborhood basis for a C^0 -topology (Note that this topology is independent of the metric chosen for P).

Let \mathcal{L} (respectively \mathcal{L}_σ) denote the set of all local homeomorphisms of M into (respectively onto) P , with induced Whitney's topology. Denote by $\nu(A)$ the cardinality of the set A , and $\infty = \nu(N)$.

Recall that $f \in \mathcal{L}_\sigma$ is said to be a *covering mapping* if for each $y \in P$ there exist a neighborhood V such that $f^{-1}(V)$ is a disjoint union of open neighborhoods W_x of $x \in f^{-1}(y)$ and $f|W_x : W_x \rightarrow V$ are homeomorphic mappings of W_x onto V for all x .

The set \mathcal{K} of all covering mappings is a disjoint union of the sets (possibly void)

$$\mathcal{K}_n = \{f \in \mathcal{L}_\sigma : \nu(f^{-1}(y)) = n, \text{ for each } y \in P\} \quad (n \in N \cup \{\infty\})$$

\mathcal{K}_n is the set of k -tuple covering mappings.

It is easy to verify that if $f \in \bigcup_{i \in N} \mathcal{K}_i$, then f is a proper mapping, i.e. $f^{-1}(F)$ is compact for every compact subset F of P .

Remark 1. The set of all proper mappings is open in \mathcal{C} [2]. Observe that, in general, the sets \mathcal{L}_σ , \mathcal{K}_1 , \mathcal{K}_2 , ... are not open in \mathcal{C} . The purpose of this paper is to prove the following

Theorem. *The sets \mathcal{L}_σ , \mathcal{K}_1 , \mathcal{K}_2 , ... are open in \mathcal{L} .*

