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Inequalities of Wirtinger's Type and Their Discrete Analogues

In this note we present some inequalities of Wirtinger type for functions defined on an interval $[0, h]$. These results have exact discrete analogues that we formulate and prove in the third section of the paper. In the fourth section we state a theorem concerning the convergence of the coefficient in a particular inequality for discrete case to the coefficient in the suitable inequality for continuous case.

The inequalities obtained in this note, as well for continuous case as for discrete one, are closely related to the boundary-value problems for differential and difference equations respectively (see [2]).

1. NOTATIONS AND PRELIMINARIES

Let R^m be the m -dimensional real Euclidean space with the usual scalar product of vectors p_1, p_2 denoted by $\langle p_1, p_2 \rangle$ and with the Euclidean norm $|p| = \sqrt{\langle p, p \rangle}$. By $L^2([0, h])$ we denote the space of all square sommable functions on $[0, h]$ with the scalar product and norm defined, respectively, by the formulae

$$\langle y_1, y_2 \rangle = \int_0^h y_1 y_2 dt, \quad \|y\| = \sqrt{\langle y, y \rangle}.$$

The set $\{0, \dots, n\}$ will be denoted by N . The difference operators $\Delta^{(k)}: R^{n+1} \rightarrow R^{n+1}$, $\nabla^{(k)}: R^{n+1} \rightarrow R^{n+1}$ for $k \in N$ are defined by the formulae

$$\Delta^{(k)}v = (\Delta^{(k)}v_0, \dots, \Delta^{(k)}v_n), \quad \nabla^{(k)}v = (\nabla^{(k)}v_0, \dots, \nabla^{(k)}v_n),$$

where for $i \in N$ we set

$$\begin{aligned} \Delta^{(0)}v_i &= v_i, & \nabla^{(0)}v_i &= v_i \\ \Delta v_i = \Delta^{(1)}v_i &= \begin{cases} v_{i+1} - v_i, & i = 0, \dots, n-1 \\ 0, & i \in n, \end{cases} & \nabla v_i = \nabla^{(1)}v_i &= \begin{cases} 0, & i = 0 \\ v_i - v_{i-1}, & i = 1, \dots, n, \end{cases} \end{aligned}$$

