

Wiesław Niedoba

A Difference Method for a Non-Linear Functional-Differential Equation of the First Order

1. In this paper we shall consider the following functional-differential equation:

$$(1.1) \quad \frac{\partial u}{\partial t} = f(t, x, u, u_x, u(t, \cdot))$$

where

$$x = (x_1, \dots, x_p), \quad u_x = \left(\frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_p} \right)$$

We construct the corresponding difference equation and we prove the convergence of the difference method in the p -dimensional case.

2. We define the following sets:

$$(2.1) \quad E = \{(t, x): 0 \leq t \leq d, 0 \leq x_j \leq \sigma, (j = 1, \dots, p)\}$$

$$(2.2) \quad P = \prod_{i=1}^p [0, \sigma]$$

$$(2.3) \quad D = \left\{ \begin{array}{l} 0 \leq t \leq d, 0 \leq x_j \leq \sigma, -\infty < u < +\infty \\ -\infty < q_j < +\infty, s \in B(P), (j = 1, \dots, p) \end{array} \right.$$

where $B(P)$ is the set of the bounded functions for $x \in P$ with the norm

$$(2.4) \quad \|s\| = \max_{x \in P} |s(x)|$$

Let us denote by m the sequence of p natural numbers

$$(2.5) \quad m = (m_1, \dots, m_p)$$

and let

$$(2.6) \quad M = (\mu, m),$$

where μ is a natural number.

We shall consider the points x^m of the real p dimensional space R^p with coordinates

$$(2.7) \quad x^m = (x_1^{m_1}, \dots, x_p^{m_p})$$

and also the nodal points

$$(2.8) \quad (t^\mu, x^m) \in R^{p+1}$$

where t^μ and x^m being defined by

$$(2.9) \quad \begin{aligned} t^\mu &= \mu \cdot k, \quad x_j^v = v \cdot h \\ \mu &= 0, 1, \dots, \nu = 0, 1, \dots, j = 1, \dots, p \end{aligned}$$

$0 < h = \text{const}, 0 < k = \text{const}.$

for $(t^\mu, x_1^{m_1}, \dots, x_p^{m_p}) \in E.$

We define the following sequences of indices

$$(2.10) \quad \omega(M) = (\mu+1, m), \quad -j(M) = (\mu, -j(m))$$

where

$$-j(m) = (m_1, \dots, m_{j-1}, m_j-1, m_{j+1}, \dots, m_p) \quad (j = 1, \dots, p)$$

Index N_h being chosen so, that

$$N_h \cdot h < \sigma \quad \text{and} \quad (N_h + 1) \cdot h \geq \sigma$$

Suppose that to each M there corresponds a number v^M . We introduce the following differences and the sum.

$$(2.11) \quad \begin{aligned} v^{M\sim} &= \frac{1}{k} (v^{\omega(M)} - v^M) \\ v^{M_j} &= \frac{1}{h} (v^M - v^{-j(M)}), \quad v^{M\Delta} = (v^{M_1}, \dots, v^{M_p}) \\ \tilde{v}^\mu(x) &= \sum_{M \in Z^\mu} v^M x^{\sim I_M}(x) \end{aligned}$$

where

$$(2.12) \quad Z^\mu = \{M: 0 \leq m_j \leq N_h, (j = 1, \dots, p)\}$$

and

$$(2.13) \quad I_M = \{x: m_i h \leq x_i < (m_i + 1) \cdot h \quad (i = 1, \dots, p)\}$$

and

$$XI_M(x) = \begin{cases} 1, & x \in I_M \\ 0, & x \notin I_M \end{cases}$$

3. In the sequel we shall use the following assumptions H .

Assumptions H

1°. Assume that the function $f(t, x, u, q, s)$, where $x = (x_1, \dots, x_p)$, $q = (q_1, \dots, q_p)$ is defined in the region D (2.3) and is of the class C^1 as the function of (t, x, u, q) and of the class C^0 as the functions of s .

