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Some Theorems on Partial Differential-Functional Inequalities of Parabolic Type

We consider a system of second order differential-functional inequalities of the type

$$(1) \quad u_i^i(t, x) \leq f^i(t, X, U, u_x^i, u_{xx}^i, U(t, \cdot)), \quad (i = 1, \dots, m),$$

where $X = (x_1, \dots, x_n)$, $U = (u^1, \dots, u^m)$, $u_x^i = (u_{x_1}^i, \dots, u_{x_n}^i)$, u_{xx}^i is the matrix of second order derivatives with respect to x and for fixed t we denote by

$$U(t, \cdot) = (u^1(t, \cdot), \dots, u^m(t, \cdot))$$

an element of the space of continuous functions such that $U(t, \cdot): X \rightarrow U(t, X)$.

Theorems 2 and 5 are generalizations of results obtained by P. Besala for parabolic differential inequalities [2], [3].

For any vectors $U = (u^1, \dots, u^m)$, $V = (v^1, \dots, v^m)$ we shall write

$$U \leq V, \text{ if } u^j \leq v^j \text{ (} j = 1, \dots, m \text{).}$$

and

$$U < V, \text{ if } u^j < v^j \text{ (} j = 1, \dots, m \text{).}$$

For a fixed i we write

$$U^i \leq V^i, \text{ if } u^j \leq v^j \text{ (} j = 1, \dots, m \text{) and } u^i = v^i.$$

A region D in the space of points (t, x_1, \dots, x_n) will be called a region of type C if the following conditions are satisfied:

(a) D is open, contained in the zone $t_0 < t < t_0 + T \leq \infty$, and the intersection of the closure of D with any closed zone $t_0 \leq t \leq t_1 < t_0 + T$ is bounded.

(b) The projection S_{t_1} on the space (x_1, \dots, x_n) of the intersection of the closure of D with the plane $t = t_1$ is, for any $t_1 \in [t_0, t_0 + T)$, non-empty.

(c) The point (t, X) being arbitrarily fixed in the closure of D , to every sequence t_v such that $t_v \in [t_0, t_0 + T)$ and $t_v \rightarrow t$, there is a sequence X_v , so that $X_v \in S_{t_v}$ and $X_v \rightarrow X$.

For a fixed $t_1 \in [t_0, t_0 + T)$ let $C_m(S_{t_1})$ stand for the space of continuous functions $z(X) = (z^1(X), \dots, z^m(X))$ from S_{t_1} in R^m with the norms

$$\|z^i\| = \max\{z^i(X) : X \in S_{t_1}\}, \quad \|z\| = \max_{1 \leq i \leq m} \|z^i\|$$

