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Remarks on a paper by Širinbekov

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In this paper we shall show the fallacy of the proofs of Theorems 1, 2 in Širinbekov's paper [2].

Let Ω be a domain in C^n and $\{u_\alpha\}$ a family of plurisubharmonic functions locally bounded from above in Ω.

Write

$$U(x) = \sup_\alpha u_\alpha(x), \qquad U^*(x) = \limsup_{x' \to x} U(x'), \qquad x \in \Omega,$$
$$N = \{x \in \Omega\colon\ U(x) < U^*(x)\}.$$

Given any complex line L (a one-dimensional complex affine subspace of C^n) suppose that $\Omega_L = \Omega \cap L$ is simply connected and define

$$V(\lambda) = \limsup_{\substack{\lambda' \to \lambda \\ \lambda' \in \Omega_L}} U(\lambda'), \qquad \lambda \in \Omega_L,$$
$$U_L = U|_{\Omega_L}, \qquad U_L^* = U^*|_{\Omega_L}, \qquad E_L' = \{\lambda \in \Omega_K\colon\ U_L(\lambda) < V(\lambda)\},$$
$$E_L = \{\lambda \in \Omega_L\colon\ V(\lambda) < U_L^*(\lambda)\}, \qquad N_L = N \cap L.$$

It follows immediately from the definitions and the well-known Cartan's Theorem (see e.g. [1], p. 107)

$$U_L \leqslant V \leqslant U_L^*, \tag{1}$$

$$N_L = E_L \cap E_L', \tag{2}$$

$$\mathrm{Cap}_2 E_L' = 0, \tag{3}$$

$$\mathrm{Cap}_2 E_L > 0 \Leftrightarrow \mathrm{Cap}_2 N_L > 0. \tag{4}$$

Given $\zeta \in \Omega_L$ the following two cases can occur:

(a) All circles $\partial B(\zeta, \delta)$ with the center ζ and a sufficiently small radius δ have non-empty intersections with $\Omega_L - E_L$.

(b) There are positive numbers r_n, $r_n \searrow 0$ such that the circles $\partial B(\zeta, r_n)$ are contained in E_L.

If (a) is satisfied then (see e.g. [1], I § 4)

$$\limsup_{\substack{\lambda \to \zeta \\ \lambda \in \Omega_L - E_L}} U_L^*(\lambda) = U_L^*(\zeta)\,.$$

Since $V(\lambda) = U_L^*(\lambda)$ in $\Omega_L - E_L$ only case (b) can occur.

According to the author of paper [2] the case (b) implies that $E_L = \Omega_L$. But this is not true.

Example. Let $\Omega = C^2$, $u_n(z_1, z_2) = \max\big(\max(0, \ln|z_1|), |z_2|^{1/n}\big)$,

$$L = \{(\lambda, 0) \in C^2 \colon \lambda \in C\}\,.$$

We see at once that

$$\Omega_L = L, \quad E_L = \{\lambda \in C\colon |\lambda| < e\}, \quad E_L' = \emptyset, \quad N_L = E_L.$$

Hence in [2] Theorems 1, 2 fail to hold true.

References

[1] Л. I. Ронкин, *Введение в теорию целых функций многих исременных*, Изд. „Наука", Москва 1971.

[2] М, Ширинбеков, *О регуляризации супремума семейства плюрисубгармонических функций*, Докл. Акад. Тадж. СССР, 1974, 17 (9)

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