

Complete lifts of tensor fields of type $(1, k)$ to natural bundles

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§ 0. Introduction and notations

Let $\pi: E \rightarrow M$ be a natural bundle. For each local diffeomorphism φ of M there is a fibred local diffeomorphism $\tilde{\varphi}$ of E such that $\tilde{\varphi}$ covers φ and the well-known conditions are satisfied (the conditions (A)–(E) in § 1). Using local 1-parameter groups of transformations we can define a mapping $\mathcal{X}(M) \rightarrow \mathcal{X}(E)$ of the Lie algebra of vector fields on M into the Lie algebra of vector fields on E . If X is a vector field on M then the image of X , denoted by X^c , is called a *complete lift* of X from M to E . The mapping

$$X \rightarrow X^c$$

is a Lie algebra homomorphism (Proposition 1.1, see also [14]). This proposition generalizes the analogous results given in [7]–[10], [18]–[22]. Here we prove this general property by a homogeneous method using the arguments of S. E. Salvioli [14].

Let t be a tensor field of type $(1, k)$ on M . We say that t admits a *complete lift* to E if there is a tensor field t^c , called a *complete lift* of t , of type $(1, k)$ on E such that

$$t^c(X_1^c, \dots, X_k^c) = (t(X_1, \dots, X_k))^c$$

for all vector fields X_1, \dots, X_k on M . In the present paper we state some properties of complete lifts of tensor fields of type $(1, k)$ and we discuss problems of the existence of a complete lift of tensor fields of type $(1, k)$ from M to E .

This definition of complete lift generalizes the definitions given by K. Yano and S. Kobayashi [20]–[22], K. Yano and S. Ishihara [18], [19] and A. Morimoto [7]–[10] in cases of natural bundles of special kinds such as tangent bundles, tangent bundles of higher order, tangent bundles of p^* -velocities and bundles of infinitesimal near points. Our definition does not coincide with that of K. Yano and E. M. Patterson [23] in the case of cotangent bundles (see (14.3) and (15.3) in [23]). We shall prove that in the case of cotangent bundles, tensor fields do not admit complete lifts in the sense of our definition (Proposition 6.2) instead of tensor fields written in the form $f\delta$, where f is a function on M .

In § 2 we give a classification of natural bundles which follows from the results of R. S. Palais and C.-L. Terng [13]. This classification (Theorem 2.1) shows that every

