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Linear Problems for Nonlinear Control Systems

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1. Introduction. In this paper we shall consider the control system

$$\frac{dx}{dt} = A(t)x+B(t)u+f(t,x,u)\,, \tag{1.1}$$

assuming the following hypothesis.

HYPOTHESIS (H). In (1.1) $x \in R^n$, $u \in R^m$, $t \in [0, T] \subset R^1$ (T is a positive number), A: $[0, T] \to R^{nxn}$ and B: $[0, T] \to R^{nxm}$ are matrices of Lebesgue summable functions and finally f: $[0, T] \times R^n \times R^m \to R^n$ is a Carathéodory mapping satisfying the growth condition

$$|f(t,x,u)| \leqslant \alpha(t)|x|+\beta(t)|u|+\gamma(t)\,. \tag{1.2}$$

Concerning the Lebesgue summable functions α, β and γ there will be some further assumptions imposed.

As usual, by a solution of (1.1) we mean an absolutely continuous function x: $[0, T] \to R^n$ provided there is an admissible control function u: $[0, T] \to R^m$ such that $x(\cdot)$ and $u(\cdot)$ satisfy (1.1) almost everywhere (a.e.) in $[0, T]$. So, in the sequel, such a pair (x, u) will also be called a solution of (1.1). By U we denote the set of admissible controls. It will be a subset of the space $L^\infty([0, T], R^m)$ of all essentially bounded functions on $[0, T]$ with values in R^m.

Given a continuous linear operator

$$L\colon C([0, T], R^n) \to R^k, \tag{1.3}$$

where $C([0, T], R^n)$ denotes the space of all continuous functions defined on $[0, T]$ and with values in R^n, we adopt the following.

DEFINITION 1.1. The control system (1.1) is said to be *L-completely controllable* in the set U iff for every $r \in R^k$ there is a solution (x, u) of (1.1), $u \in U$ such that $Lx = r$.

One of the purposes of this paper is to establish the conditions under which the L-complete controllability of the linear system

$$\frac{dx}{dt} = A(t)x+B(t)u \tag{1.4}$$

implies the L-complete controllability of the perturbed system (1.1). The other purpose is to look for some generic properties like openess or density of the set of operators L for which system (1.4) is L-completely controllable (or similar problem for the set of equations provided L is fixed).

The paper consists of five sections. After preliminaries given in Section 2 we prove, in Section 3, two lemmae concerning the characterization of linear systems which are L-completely controllable. A solution of the first of two mentioned above problems is proposed in Section 4, see Theorem 4.1 whose proof is based on the Schauder fix-point theorem. The second problem is worked out in Section 5. In particular, the proof of Theorem 5.1 goes by an argument similar to that used by Shui-Nee Chow and A. Lasota in paper [4].

Remark 1.1. Setting $k = 2n$, $Lx = (x(0), x(T))$ in Definition 1.1 we get the classical notion of the complete controllability (see [8]). In this case a theory concerning the first problem mentioned above was developed in a series of papers [9], [1], [2], [5], [6], [3]. The L-complete controllability with

$$Lx = Mx(\tau)+Nx(\tau)+\int_{\tau}^{T} dF(\theta)x(\theta),$$

where M, N are constant matrices and $F(\cdot)$ is a matrix of functions with bounded variation, was considered by C. Marchio [10] but only for linear system (1.4). There are also some results concerning the so-called controllability in direction p [7]. The last case is also included in our theory, if we put $k = n+1$, $Lx = (x(0), p^*x(T))$, where p^* denotes the transpose of vector p. Thus our "controllability" results can be regarded as an extension of the previous ones to the case of general operator L and to perturbations f without the Lipschitz condition.

2. Notation and preliminaries. As usual, by $x = (x_1, \ldots, x_n)$ we denote a vector of the n-dimensional Euclidean space R^n. The symbols $|\cdot|$ and $\langle\cdot,\cdot\rangle$ denote, respectively, a norm and the scalar product in a finite dimensional space, while in the functional spaces $C([0, T], R^n)$, $L^p([0, T], R^n)$, $1 \leqslant p \leqslant \infty$, the norm will be denoted, respectively, by $\|\cdot\|$ and $\|\cdot\|_p$. By D^* we denote the transpose of a metrix D.

We recall that given a control $u \in U$ and $x^0 \in R^n$ we obtain the solution of the linear system (1.4) by the variation of parameters formula:

$$x(t) = X(t)x^0+\int_0^t X(t)X^{-1}(s)B(s)u(s)\,ds, \tag{2.1}$$

where $X(t)$ denotes the fundamental matrix solution of the system $\frac{dx}{dt} = A(t)x$ (i.e. $\frac{dX}{dt} = A(t)X(t)$, $X(0) = I =$ identity matrix). Setting

$$K(t, s) = \begin{cases} X(t)X^{-1}(s) & \text{if } 0 \leqslant s \leqslant t \\ 0 & \text{if } t < s \leqslant T \end{cases} \tag{2.2}$$

we can write (2.1) in the form

$$x(t) = X(t)x^0 + \int_0^T K(t, s)B(s)u(s)ds. \tag{2.3}$$

Let us notice that there exists a constant $l>0$ such that

$$|K(t, s)| \leqslant l \quad \text{for every } (t, s) \in [0, T] \times [0, T]. \tag{2.4}$$

So the mappings

$$[0, T] \ni s \to K(t, s)B(s)u(s) \in R^n, \qquad t \in [0, T]$$

are measurable and uniformly bounded by a summable function and the mapping

$$[0, T] \ni t \to \int_0^T K(t, s)B(s)u(s)ds \in R^n$$

is absolutely continuous. Since the operator L given by (1.3) can be represented in the form

$$Lx = [(Lx)_1, \ldots, (Lx)_k]^*, \ (Lx)_i = \sum_{i=1}^{n} \int_0^T x_j(t)d\mu_{ij}(t) \tag{2.5}$$

we obtain from (2.3), by Fubini theorem,

$$Lx = L[X(\cdot)x^0] + \int_0^T L[K(\cdot, s)B(s)u(s)]ds. \tag{2.6}$$

In the last term of the above formula L means the extension given by (2.5) (with the same Radon measures μ_{ij}) of the operator L defined by (1.3) to the space $L^\infty([0, T], R^n)$. By a direct calculation one can prove that formula (2.6) may be written in the form

$$Lx = L^n[X(\cdot)]x^0 + \int_0^T L^n[K(\cdot, s)]B(s)u(s)ds, \tag{2.7}$$

where L^n denotes the mapping induced by L which to every $(n \times n)$ matrix Y of functions assigns the $(k \times n)$ matrix of constants obtained by the application of L to every column of Y (i.e. if

$$Y(\cdot) = \{y_{ij}(\cdot)\}_{ij=1,\ldots,n}, \qquad \text{then } L^n[Y(\cdot)] = \{L[y_{i1}(\cdot)]_{i=1,\ldots,n}, \ldots, L[y_{i,n}(\cdot)]_{i=1,\ldots,n}\}).$$

Since the set {columns of $K(\cdot, s)$: $s \in [0, T]$} is bounded in $L^\infty([0, T], R^n)$ and L is a finite dimensional operator (with values in R^k), there exists a constant $\tilde{a}>0$ such that

$$\sup\{|L^n[K(\cdot, s)]| : s \in [0, T]\} \leqslant \tilde{a}. \tag{2.8}$$

3. *L*-controllability for linear problems. Let us assume that the class U of admissible controls is the space $L^\infty([0, T], R^m)$ and define the map Φ: $R^n \times U \to R^k$ setting

$$\Phi(x^0, u) = Lx, \text{ where } Lx \text{ is given by (2.7)}. \tag{3.1}$$

It is clearly seen that the system (1.4) is L-completely controllable in U iff the linear mapping Φ is surjective which in turn is equivalent to the fact that the conjugate mapping $\Phi^*\colon R^k \to R^n \times U^*$ is injective (U^* being the dual space of U). Hence, and from the formula $(\Phi^* y)(x^0, u) = \langle \Phi(x^0, u), y\rangle = \langle x^0, L^n[X(\cdot)]^* y\rangle + \int_0^T \langle u(s), B^*(s) L^n[K(\cdot, s)]^* y\rangle ds$, ($y \in R^k$) we get the following result of Marchio [10]

LEMMA 3.1. *System* (1.4) *is L-completely controllable in $L^\infty([0, T], R^m)$ iff the following implication holds*

$$\left.\begin{cases} y \in R^k, L^n[X(\cdot)]^* y = 0 \\ B(s)^* L^n[K(\cdot, s)]^* y = 0 \text{ a.e. in } [0, T] \end{cases}\right\} \Rightarrow y = 0. \tag{3.2}$$

Now, setting

$$\varphi(\tau) = \int_\tau^T L^n[K(\cdot, s)] \cdot B(s) ds \tag{3.3}$$

$$M = L^n[X(\cdot)] L^n[X(\cdot)]^* + \int_0^T \varphi(\tau)\varphi(\tau)^* d\tau \tag{3.4}$$

we get the formula

$$\langle y, My\rangle = |L^n[X(\cdot)]^* y|^2 + \int_0^T |\varphi(\tau)^* y|^2 d\tau \tag{3.5}$$

and we can formulate the following

LEMMA 3.2. *System* (1.4) *is L-completely controllable in $L^\infty([0, T], R^m)$ iff the matrix M given by* (3.4) *is positive definite. Moreover, in this case system* (1.4) *is L-completely controllable in any set U containing all controls of the class $U_M = \{u_y(\cdot)\colon y \in R^k\}$, where $u_y(\cdot)$ is given by*

$$u_y(t) = \int_0^t \varphi(\tau)^* M^{-1} d\tau y. \tag{3.6}$$

Proof. The first part of the lemma follows from (3.5) and Lemma 3.1. For the second assertion notice that any point $y \in R^k$ can be reached by means of control $u_y(\cdot)$ given by (3.6) if we start from initial point x_y^0 defined by

$$x_y^0 = L^n[X(\cdot)]^* M^{-1} y. \tag{3.7}$$

Indeed, owing to the integration by parts formula we have

$$\Phi(x_y^0, u_y) = L^n[X(\cdot)] L^n[X(\cdot)]^* M^{-1} y + \int_0^T \int_\tau^T L^n[K(\cdot, s)] B(s) ds \varphi(\tau)^* M^{-1} y d\tau$$
$$= MM^{-1} y = y,$$

what completes the proof.

Remark 3.1. It is easily seen from the definitions that there exist positive constants $\tilde{b}$, $\tilde{d}$, $\tilde{e}$ and $\tilde{h}$ such that the following estimates hold: $\max_{\tau\in[0,T]} |\varphi(\tau)| \leqslant \tilde{b}$, $|M| \leqslant \tilde{d}$, $|u_y(t)| \leqslant \tilde{e}|y|t$, $|x_y^0| \leqslant \tilde{h}|y|$.

Remark 3.2. Let us notice that each control of the class U_M defined above has the absolutely continuous derivative. It is worth observing (the same reasoning) that the regularity of controls could be augmented arbitrarily and the Lemma 3.2 will still hold. For instance, replacing φ and M, by $\tilde{\varphi}$ and $\tilde{M}$ defined, respectively, by the formulae:

$$\tilde{\varphi}(s) = \int_0^s \varphi(\tau)d\tau, \qquad \varphi(\tau) \text{ given by (3.3)}$$

$$\tilde{M} = L^n[X(\cdot)]L^n[X(\cdot)]^* + \int_0^T \tilde{\varphi}(s)\tilde{\varphi}(s)^* ds,$$

we get the class $U_{\tilde{M}} = \{\tilde{u}_y(\cdot)\colon y \in R^k\}$ of controls

$$\tilde{u}_y(t) = \int_0^t \int_\sigma^T \tilde{\varphi}(s)^* ds\, d\sigma\, \tilde{M}^{-1} y$$

which have the third derivative absolutely continuous. One can check directly that $\Phi(\tilde{x}_y^0, \tilde{u}_y) = y$ for every $y \in R^k$ if we put $\tilde{x}_y^0 = L^n[X(\cdot)]^* \tilde{M}^{-1} y$.

4. *L*-controllability of nonlinear systems. We start with the auxiliary problem:

$$\left\{\begin{aligned} &\frac{dx}{dt} = A(t)x + B(t)u + c(t), \\ &Lx = r, \end{aligned}\right. \tag{4.1}$$

where $c \in L^1([0, T], R^n)$, $r \in R^k$. By the straightforward calculation one can prove the following

LEMMA 4.1. *If we put*

$$y_c = r - \int_0^T L^n[K(\cdot, s)]c(s)ds \tag{4.2}$$

and define u_{y_c} *and* $x_{y_c}^0$, *respectively, by the formulae* (3.6) *and* (3.7) (*with* y *replaced by* y_c), *then* x_c *given by*

$$x_c(t) = X(t)x_{y_c}^0 + \int_0^T K(t, s)B(s)u_{y_c}(s)ds + \int_0^T K(t, s)c(s)ds \tag{4.3}$$

is a solution of problem (4.1).

Remark 4.1. It is clearly observed that if we fix r, then the affine mapping $G\colon L^1([0,T],R^n)\to C([0,T],R^n)\times L^\infty([0,T],R^m)$ which assigns to function c the pair (x_c, u_{y_c}) defined by Lemma 4.1 is continuous and we have the estimation

$$\|G(c)\| = \|(x_c, u_{y_c})\| := \|x_c\|+\|u_{y_c}\|_\infty \leqslant a\|c\|_1+b \tag{4.4}$$

with some positive constants a and b.

In fact, by (2.8) and Remark 3.1 we get

$$|y_c|\leqslant|r|+\tilde{a}\|c\|_1, \tag{4.5}$$

$$\|u_{y_c}\|_\infty\leqslant\tilde{e}T|y_c|\leqslant\tilde{e}T(|r|+\tilde{a}\|c\|_1), \tag{4.6}$$

$$|x^0_{y_c}|\leqslant\tilde{h}|y_c|\leqslant\tilde{h}(|r|+\tilde{a}\|c\|_1). \tag{4.7}$$

So from (4.3) we obtain by (2.4)

$$|x_c(t)|\leqslant X(t)\tilde{h}(|r|+\tilde{a}\|c\|_1)+l\|B(s)\|_1\tilde{e}T(|r|+\tilde{a}\|c\|_1)+l\|c\|_1$$

and in consequence we have (4.4) with

$$a = \tilde{a}\tilde{h}\max_{t\in[0,T]}|X(t)|+l\,\tilde{e}\,T\tilde{a}\|B(\cdot)\|_1+l+\tilde{e}\,\tilde{a}\,T, \text{ and}$$

$$b = \tilde{h}|r|\max_{t\in[0,T]}|X(t)|+l\,T\,\tilde{e}|r|\|B(\cdot)\|_1+\tilde{e}\,T|r|.$$

Let us now consider the problem:

$$\begin{cases}\dfrac{dx}{dt} = A(t)x+B(t)u+f(t,x,u)\\ Lx = r\end{cases} \tag{4.8}$$

assuming Hypothesis (H) from Introduction. The existence of solution of (4.8) follows from the Schauder fix-point theorem applied to the map $F\colon C([0,T],R^n)\times U_M\to C([0,T],R^n)\times U_M$ which to a pair (z,v) assigns the pair (x_c, u_{y_c}) given by Lemma 4.1 with $c(t)=f(t,z(t),v(t))$; i.e. $F(z,v)=(x_c,u_{y_c})$. Thus it remains to check that F is continuous, compact and maps a closed ball into itself.

To prove the continuity of F notice that $F=G\circ H$, where the continuous mapping G is defined in Remark 4.1 while the continuity of mapping

$$H\colon C([0,T],R^n)\times U_M\ni(z,v)\to c(t):=f(t,z(t),v(t))\in L^1([0,T],R^n)$$

follows (by Lebesgue theorem) from the assumption concerning f (see Hypothesis (H)).

Now, setting $B_\varrho=\{(z,v)\in C([0,T],R^n)\times U_M\colon \|z\|+\|v\|_\infty\leqslant\varrho\}$ we prove that there exists $\varrho_0>0$ such that

$$F(B_\varrho)\subset B_\varrho \quad \text{for } \varrho\geqslant\varrho_0, \tag{4.9}$$

provided $\delta=\|\alpha\|_1+\|\beta\|_1$ is sufficiently small. To this end we observe that for $(z,v)\in B_\varrho$ we have

$$\|H(z,v)\|_1 = \|f(t,z(t),v(t))\|_1 \leqslant\varrho(\|\alpha\|_1+\|\beta\|_1)+\|\gamma\|_1. \tag{4.10}$$

Hence and from (4.4) with $c(t) = f(t, z(t), v(t))$ we obtain the estimation

$$\|F(z, v)\|_* = \|G \circ H(z, v)\|_* \leqslant a\varrho(\|\alpha\|_1 + \|\beta\|_1) + a\|\gamma\|_1 + b, \quad (z, v) \in B_\varrho,$$
$$(\|(z, v)\|_* = \|z\| + \|v\|_\infty),$$

which implies (4.9) with $\delta = \|\alpha\|_1 + \|\beta\|_1 < \frac{1}{a}$.

Finally, the compactness of F is a consequence of mentioned below properties of the following operators:

(i) H is bounded on B_ϱ (see (4.10)),

(ii) $L^1 \ni c \to y_c \in R^k$ (y_c given by (4.2)) is compact as an affine finite dimensional operator,

(iii) $R^k \ni y_c \to x^0_{y_c} \in R^n$ and $R^k \ni y_c \to u_{y_c} \in L^\infty([0, T], R^m)$ ($x^0_{y_c}$ and u_{y_c} given, respect, by (3.7) and (3.6)) are compact as bounded and finite dimensional operators,

(iv) $L^1([0, T], R^n) \ni c(\cdot) \to B(\cdot)u_{y_c}(\cdot) + c(\cdot) \in L^1([0, T], R^n)$ is bounded by (4.6),

(v) $L^1([0, T], R^n) \ni w(\cdot) \to \int_0^T K(\cdot, s)w(s)ds \in C([0,T], R^n)$ is compact owing to (2.4).

Thus, we have proved the following

THEOREM 4.1. *If the linear system* (1.4) *is L-completely controllable in* $L^\infty([0, T], R^m)$, *then the perturbed system* (1.1) *is also L-completely controllable in any set* $\tilde{U}$ *containing the set* U_M *of controls given by* (3.6) *provided the Hypothesis* (H) *is satisfied with* α *and* β *sufficiently small in the sense that there exists a constant* $\delta > 0$ *such that* $\|\alpha\|_1 + \|\beta\|_1 \leqslant \delta$.

5. Generic properties of L-controllable systems. In this section we assume the control system

$$\frac{dx}{dt} = A(t)x + B(t)u + c(t) \tag{5.1}$$

(A, B, c are as in problem (4.1)) is fixed. We ask the question: How big is the set $\mathscr{L}_0$ of all linear and continuous operators L from $C([0, T], R^n)$ into R^k for which the system (5.1) is L-completely controllable in $U = L^\infty([0, T], R^m)$ Setting

$$Lx = r \tag{5.2}$$

and $\mathscr{L}(C, R^k) = \mathscr{L}(C([0, T], R^n), R^k)$ for simplicity sake we can write down the set $\mathscr{L}_0$ as follows:

$$\mathscr{L}_0 = \{L \in \mathscr{L}(C, R^k)\colon \text{the problem (5.1) (5.2) has a solution for every } r \in R^k\}.$$

THEOREM 5.1. *The set* $\mathscr{L}_0$ *is open and dense in* $\mathscr{L}(C, R^k)$ *(with respect to the norm topology) provided it is nonempty.*

Proof. First, let us notice that the subset $\mathcal{M}^p_{k\times k}$ of all positive definite matrices is open in the set $\mathcal{M}_{k\times k}$ of all $(k\times k)$ constant matrices. Then we observe that the mapping

$$\Psi\colon \mathcal{L}(C, R^k)\ni L\to M_L\in\mathcal{M}_{k\times k}, \quad \text{with } M_L = M \text{ given by (3.4)},$$

is continuous. In consequence $\mathcal{L}_0 = \psi^{-1}(\mathcal{M}^p_{k\times k})$ is open. To prove the density part of the theorem we have to show that for every $\varepsilon>0$ and $L\in\mathcal{L}(C, R^k)$ there exists $L_1\in\mathcal{L}_0$ such that

$$\text{(5.3)} \qquad |||L_1-L|||<\varepsilon \quad (|||\cdot||| \text{ denotes the norm in } \mathcal{L}(C, R^k)).$$

To this end notice first that by the assumption of the theorem there exists $L_0\in\mathcal{L}_0$. Given L and L_0 we observe that by (3.4) and (3.3) we have

$$M_{\lambda L_0+L} = \lambda^2 M_{L_0}+\lambda N(L_0, L)+M_L \qquad \text{for every } \lambda\in R,$$

where

$$N(L_0, L) = L_0^n[X(\cdot)]L^n[X(\cdot)]^*+L^n[X(\cdot)]L_0^n[X(\cdot)]^*+\int_0^T \varphi_{L_0}(\tau)\varphi_L(\tau)^*+\varphi_L(\tau)\varphi_{L_0}(\tau)^*d\tau.$$

So the function $\varrho\colon R\to R$ given by

$$\text{(5.4)} \qquad \varrho(\lambda) = \det M_{\lambda L_0+L} \quad (\det = \text{determinant})$$

is analytic. Moreover, since by Lemma 3.2 M_{L_0} is positive definite we have

$$\frac{\varrho(\lambda)}{\lambda^{2k}} \neq 0 \quad \text{for sufficiently large } \lambda.$$

Hence, $\varrho(\lambda)$ does not vanish identically in a neighbourhood of 0. Thus there exists a sequence $\{\lambda_n\}\subset R^1$ such that

$$\text{(5.5)} \qquad \varrho(\lambda_n)\neq 0 \quad \text{and } \lambda_n\to 0 \quad \text{as } n\to\infty.$$

Hence, we can conclude that for every n, $M_{\lambda_n L_0+L}$ is positive definite (it is symmetric and positive semi-definite by definition). In consequence, by Lemma 3.2 we find that $\lambda_n L_0+L\in\mathcal{L}_0$ for every n. So we have (5.3) with $L_1 = \lambda_n L_0+L$ for sufficiently large n, and the proof of the theorem is completed.

It is clearly observed that if we fix L, A, B and we define perturbation f by the formula

$$\text{(5.6)} \qquad f(t, x, u) = A_0(t)x+B_0(t)u+c(t),$$

where A, B, A_0, B_0 are as in Hypothesis (H), $c\in L^1([0, T], R^n)$ and L is given by (1.3), then from Theorem 4.1 we obtain

Corollary 5.1. *If system* (1.4) *is L-completely controllable in* $U = L^\infty([0, T], R^m)$, *then perturbed system* (1.1) *with f given by* (5.6) *is also L-completely controllable in U provided the function matrices* $A_0(\cdot)$ *and* $B_0(\cdot)$ *are sufficiently small in* $L^1([0, T], R^{n\times n})$ *and* $L^1([0, T], R^{n\times m})$, *respectively. So the set of all Lebesgue summable matrices* $A(\cdot)$, $B(\cdot)$ *for which the system* (5.1) *is L-completely controllable in U is open with respect to* L_1*-norm topology in the set of all Lebesgue summable matrices as in Hypothesis* (H).

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