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Canonical Extension of Local Pseudo-dynamical Semi-systems

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In the present paper we construct an extension of local pseudo-dynamical semi-systems $(X, G_*, A; \pi)$, where G_* is a positive semigroup of an ordered group G, to pseudo-dynamical systems, with values which are non-empty subsets of the space X. The extension considered is so far minimal that the emission zone of a point in the extended system gives the minimal set, which together with its supplement is a positive invariant in the sense of positive invariability in the $(X, G_*, A; \pi)$ semi-system.

We use the terminology for local pseudo-dynamical semi-system and for its generalization introduced by A. Pelczar in [1]. According to this terminology the word "local" refers to the domain of π only a subset $A \subset G \times X$, "pseudo" to the lack of continuity of π, and the word "semi" to the fact that $(G, +)$ is a semigroup.

The author introduces the notion of canonical extension with regard to the equivalence relation in X, compatible in a sense with the semi-system, and inducing a transformation of the extended system with values in equivalence classes with regard to the relation considered.

1. Let X be a non-empty set henceforth called a space $(G, +)$ an Abel semigroup with neutral element 0, A a non-empty subset of $G \times X$ such that $\{0\} \times X \subset A$. For $x \in X$ let us put

$$I_x := \{t \in G\colon\ (t, x) \in A\}.$$

DEFINITION 1. (Def. 1.2.1 in [1]). Let π be a transformation in A with values in X. The quadruple $(X, G, A; \pi)$ is called a *local pseudo-dynamical semi-system* (abbr. Loc PDS-*system*) if and only if the following conditions are satisfied:

(i) $\pi(0, x) = x$ for every $x \in X$,

(ii) if $x \in X$, $t \in I_x$, $s \in I_{\pi(t,x)}$ then $t+s \in I_x$,

(iii) for $x \in X$, $t \in I_x$, $s \in I_{\pi(t,x)}$ we have

$$\pi(s, \pi(t, x)) = \pi(s+t, x).$$

If $(G, +)$ is a group and if

(iv) for $x \in X$, $t \in I_x$ we have $-t \in I_{\pi(t,x)}$

then $(X, G, A; \pi)$ is called a *local pseudo-dynamical system* (abbr. Loc. PD-*system*).

DEFINITION 2. (Def. 1.7.1 in [1]). Let λ be a transformation on A (A as before) with values in the set of all non-empty subsets of X. The quadruple $(X, G, A; \lambda)$ is called a *generalized local pseudo-dynamical semi-system* (abbr. G Loc PDS-*system*) if and only if the following conditions are fulfilled:

(j) $\lambda(0, x) = \{x\}$ for $x \in X$,

(jj) if $x \in X$, $t \in I_x$, $y \in \lambda(t, x)$, $s \in I_y$ then $t+s \in I_x$,

(jjj) for $x \in X$, $t \in I_x$, $y \in \lambda(t, x)$, $s \in I_y$, we have

$$\lambda(s, y) \subset \lambda(t+s, x) .$$

DEFINITION 2'. Let λ be a transformation on A (A as before) with values in the set of all non-empty subsets of X. The quadruple $(X, G, A; \lambda)$ is called a *multivalued local pseudo-dynamical semi-system* (abbr. M Loc PDS-*system*) if and only if the following conditions are fulfilled:

(j') $x \in \lambda(0, x)$ for $x \in X$,

(jj) and (jjj) as before.

Similarly a MLocPDS-system is called a *multivalued local pseudo-dynamical system* (abbr. MLocPD-*system*) if and only if $(G, +)$ is a group and if the following condition holds:

(jv) if $x \in X$, $t \in I_x$, $y \in \lambda(t, x)$, then $-t \in I_y$ and $x \in \lambda(-t, y)$.

Note that if $(X, G, A; \lambda)$ is a GLocPDS-system, $A = G \times X$ and G is a group, then for every $(t, x) \in G \times X$ the set $\lambda(t, x)$ contains only one element. This follows from the inclusion:

$$\lambda(t, x) \subset \lambda(t+(-t), y) \qquad \text{for } y \in \lambda(-t, x) .$$

Then a GLocPDS-system may be regarded as a LocPD-system. This conclusion is not true in the case of an MLocPDS-system.

Remark 1. Let $(X, G, A; \lambda)$ be an MLocPDS-system. Then

1) the relation

$$yR_0x \overset{\text{df}}{\Leftrightarrow} y \in \lambda(0, x)$$

is reflexive and transitive,

2) if $z \in \lambda(t, x)$, yR_0z, then $y \in \lambda(t, x)$.

Example 1. Let $X = \boldsymbol{R}^2$ $(\boldsymbol{R} = (-\infty, \infty))$, $(G, +) = (\boldsymbol{R}_*, +)$ $(\boldsymbol{R}_* = [0, \infty))$. Define the transformation λ in $\boldsymbol{R}_* \times \boldsymbol{R}^2$ by the formula

$$\lambda(t, (x, y)) := \{x+t\} \times [y-t, y+t] .$$

In $\boldsymbol{R} \times \boldsymbol{R}^2$ let us put

$$\mu(t, (x, y)) := \{x+t\} \times (-\infty, y] .$$

Then $(\boldsymbol{R}^2, \boldsymbol{R}_*, \boldsymbol{R}_* \times \boldsymbol{R}^2; \lambda)$ is a GLocPDS-system and $(\boldsymbol{R}^2, \boldsymbol{R}, \boldsymbol{R} \times \boldsymbol{R}^2; \mu)$ is an MLoc PDS-system.