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CHAOS IN THE LORENZ EQUATIONS FOR CLASSICAL PARAMETER VALUES. A COMPUTER ASSISTED PROOF

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The aim of this note is to announce the computer assited proof of chaos for the Lorenz systems with classical parameter values (see [2] for a full version). By chaos we mean here the existence of symbolic dynamics for some iterate of suitable Poincaré map and of infinitely many periodic points with increasing periods.

The method used in the proof developed by the second author in [7] and [8] is based on the notion of TS-maps. This method was applied previously to the Rössler equations, the Hénon map and to Chua's circuit [1].

The idea of computer assisted proof of chaotic dynamics based on topological invariants appeared first in the work of Mischaikow and Mrozek ([4], [5]). They used the discrete Conley index introduced in [6] to prove the existence of chaotic dynamics for the Hénon map and the Lorenz equations (nonclassical parameter values). We believe that our method is considerably easier to understand and apply — the topological machinery behind the fixed point index (involved in the definition of TS-maps) is relatively simple when compared to the one involved in the discrete Conley index.

The Lorenz equations are given by [3]

$$
\begin{aligned}
\dot{x} &= s(y-x), \\
\dot{y} &= rx - y - xz, \\
\dot{z} &= xy - qz,
\end{aligned}
\tag{1}
$$

where $s = 10$, $r = 28$, $q = 8/3$.

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Let us set $\Sigma = \{(x,y,z) \in \mathbf{R}^3 \colon z = r-1\}$. This is a standard choice for the Poincaré section. Let $\mathbf{P}$ be a Poincaré map generated on the plane Σ.

The main result of our work is

THEOREM 1. *For all parameter values in a sufficiently small neighbourhood of $(s,r,q) = (10, 28, 8/3)$ there exists a transversal section $I \subset \{z = 27\}$ such that the Poincaré map $\mathbf{P}$ induced by (1) is well defined and continuous on I. There exists a continuous surjective map $\pi : \mathrm{Inv}(I, \mathbf{P}^2) \to \Sigma_2$, such that*

$$\pi \circ \mathbf{P}^2 = \sigma \circ \pi.$$

The preimage of any periodic sequence from Σ_2 contains periodic points of $\mathbf{P}^2$.

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